Audio Engineering Society

Convention Paper

Presented at the 117th Convention
2004 October 28–31 San Francisco, CA, USA

Raised Cosine Equalization Utilizing Log Scale Filter Synthesis

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ABSTRACT

An improved method of audio equalization utilizing Raised Cosine Filters is introduced. Raised Cosine Filters offer improved selectivity in comparison to traditionally implemented equalization functions, while also maintaining beneficial attributes such as a minimum phase response. The Raised Cosine Filter also enables flat summation and asymmetrical filtering characteristics, resulting in an equalization system offering capability beyond traditional filter implementations.

1. INTRODUCTION

The introduction of parametric equalization provided new opportunities for audio engineers [1]. The ability to continuously adjust filter center frequency, bandwidth and amplitude freed engineers from the constraints of frequency selective EQ’s with fixed shape and stepped frequency selection. Since the introduction of the parametric equalizer in the early 1970s however, the audio practitioner’s “tool set” has largely remained the same.

The traditional parametric audio filter provides a symmetrical frequency response function when plotted on a logarithmically spaced frequency axis. This parametric function is well known and ubiquitous. In most applications however, the parametric function is non-ideal. The parametric function provides useful selectivity that can be adjusted by a Q or bandwidth parameter, but the filter “skirts” continue to affect frequencies well beyond the Q or bandwidth specification. Sonically, this “out of band frequency emphasis” results in users aiming for one result and receiving unwanted changes to neighboring frequencies. These skirts, inherent to the traditional parametric filter shape, result in non-intuitive control.

Whether parametric filters are implemented using analog or digital techniques, the theoretical filter shapes are identical. To create the equalization shapes required for optimization, users are forced to stack up a number of parametric sections to create a piecewise approximation to the desired response. Using newly developed filter synthesis techniques, it is now possible to create new and more practical filter shapes.

This paper describes a new filter type called the Raised Cosine Filter. The Raised Cosine Filter provides an improved method of audio equalization. The filter type provides a frequency response shape that is of higher selectivity when compared to a traditionally implemented analog or digital filter. Audibly, users are able to adjust a particular frequency range with improved precision; Neighboring frequency ranges remain unaffected. The paper introduces the Raised Cosine Filter, compares the new filter to traditional filter implementations, and discusses applications in which the Raised Cosine Filter offers improvements over traditional filtering techniques.

Raised Cosine equalization is made possible through an advanced filtering technique that synthesizes equalization coefficients on a logarithmic scale. The underlying technology affords a constant 1/24th octave filter bank throughout the Nyquist interval. The focus of this paper is the application of Log Scale Filter Synthesis to provide Raised Cosine equalization.

In many respects, the Raised Cosine Filter simply provides a higher order filter function. We will compare the frequency response shape of the traditional parametric filter and the Raised Cosine Filter. This comparison will show the higher selectivity of the Raised Cosine, and it will show that it requires a filter bank of 7 traditional parametric filters to match the response of a single Raised Cosine function to an acceptable tolerance.

Within this analysis, we will also discuss the phase response of the Raised Cosine Filter, since this is an important attribute in real world applications such as optimizing loudspeaker/room interaction. The magnitude and phase response of the filter bank of traditionally implemented parametric filters and a single Raised Cosine Filter will be shown to be identical.

The paper will also discuss the beneficial attributes of minimum phase equalization for correcting loudspeaker responses in acoustic spaces. Measurements of a typical loudspeaker have been acquired in an anechoic chamber. A complex reflector has been placed in the chamber to provide a real-world reflection source. It will be shown that the application of a Raised Cosine Filter with a minimum phase response is capable of correcting for this acoustic reflection in the resulting frequency response.

The Raised Cosine Filter also enables some new and interesting capabilities, above and beyond what a traditional parametric filter can achieve. Because of the Raised Cosine shape, neighboring filters can be configured to sum to a flat frequency response magnitude. This is of particular interest to applications requiring graphic equalizers. In the paper we will show how traditionally implemented graphic equalizers (using a third octave graphic EQ as an example) suffer from out of band frequency emphasis resulting in a frequency response which deviates from what a user would expect from the position of the controls. The conclusion is that the Raised Cosine Filter offers the first truly graphic equalizer: an equalizer whose control settings exactly match the resulting frequency response transfer function.

The Raised Cosine Filter can be further manipulated by splitting the two filter halves with a flat transition in between. The resulting filter shape no longer resembles the familiar bell curve of a traditional parametric equalizer or the new higher selectivity shape of the Raised Cosine, rather, the shape resembles a flat topped mesa (the familiar geological erosion feature common in the United States Southwest). This new equalizer shaping tool adds new degrees of freedom to the traditional parametric equalizer: two center frequencies, shape and amplitude.

Additionally, the Raised Cosine function enables asymmetrical filtering capabilities. The Raised Cosine function can be configured to provide a filter in which each side of the normally symmetrical bell shape can be adjusted independently. This results in two center frequencies and two bandwidth values for adjusting each filter transition edge independently. This asymmetrical filter response provides an improved match to the high order frequency response variations commonly associated with electrodynamic loudspeaker systems.

2. RAISED COSINE FILTER

Utilizing the functionality provided by Log Scale Filter Synthesis, extensions to the traditional parametric shape are possible. Instead of being limited to the frequency response shape defined by a second order filter section, we searched for a more ideal function. We were looking for a shape that provided higher selectivity, a well defined bandwidth specification, and a magnitude response that summed to flat with neighboring filters. The Raised Cosine Filter shape was defined based upon these attributes.

We start the analysis of the raised cosine shape by looking at the simplest function, the shelf filter. The figure below depicts an example shelf function displayed on a logarithmic frequency axis:

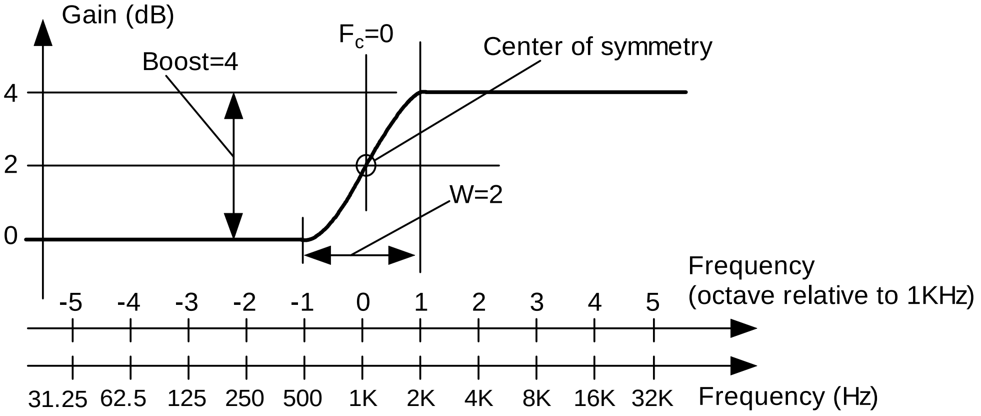

Figure 1: Raised Cosine Shelf Filter

Where:

Fc : Center frequency (in octaves relative to 1kHz)

W : Width (in octaves)

$Boost$: Gain in pass-band (dB)

In this example, the shelf filter transitions from 0 to +4dB, with the +2dB center point being at *Freq*=1 kHz. For convenience, we calculate the curve in terms of a logarithmic frequency scale f, defined as:

$$f = \log_2\left(\frac{Freq}{1000}\right)$$

so that the linear scale (*Freq (Hz)*) is converted to the logarithmic scale (*f (octaves)*).

Our example filter has a center frequency of 1 kHz, with its transition width extending from 500 Hz to 2 kHz (from *f*=-1 to *f*=+1 octaves, relative to 1kHz). The high frequency gain is +4dB.
We choose the shape of the shelf filter based on the following criteria:

- We desire that the transition band of the shelf filter should be clearly constrained, so that the gain of the filter below 500 Hz should be 0 dB, and the gain of the filter above 2 kHz should be 4 dB
- The transition region should smoothly rise from 0 dB (at 500 Hz) to +4 dB (at 2 kHz)
- The slope of the filter should be continuous (so there should be no sharp corners on the curve)
- The filter should be symmetric (note the center of symmetry shown in the diagram).

Many different curves will fit these criteria, but a half-cycle cosine is one of the most attractive choices due to secondary benefits of the shape for this application. This gives us the following equations for the gain of our shelf filter (gain, in dB, as a function of the logarithmic frequency, *f*, in octaves):

$$Gain(f) = \begin{cases} 0 & \left(f \le F_c - {}^{W}\!/_{2}\right) \\ Boost \times \left\{ 0.5 + 0.5 \times \cos\left(\pi \dfrac{f - F_c - {}^{W}\!/_{2}}{W} \right) \right\} & \left(F_c - {}^{W}\!/_{2} < f < F_c + {}^{W}\!/_{2}\right) \\ Boost & \left(F_c + {}^{W}\!/_{2} \le f\right) \end{cases}$$

We define a parametric filter section by creating it from two shelf segments (a rising shelf followed immediately by a falling shelf).

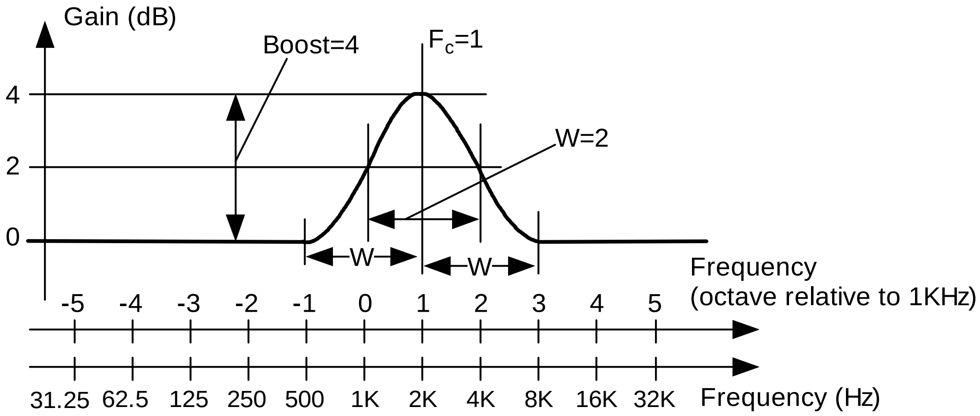

Figure 2: Raised Cosine Parametric Filter

Where:

Fc :	Center frequency (in octaves relative to 1kHz)
W :	Width (in octaves)
$Boost$:	Gain in pass-band (dB)

$$Gain(f) = \begin{cases} Boost \times \left\{ 0.5 + 0.5 \times \cos\left(\pi \dfrac{f - F_c}{W} \right) \right\} & \left(\left| f - F_c \right| < W\right) \\ 0 & (otherwise) \end{cases}$$

3. COMPARISON OF RAISED COSINE FILTER TO TRADITIONAL PARAMETRIC

3.1. Magnitude Comparison

Figure 3 below shows two actual measured transfer functions comparing a third-octave Raised Cosine parametric filter and a third-octave traditionally implemented parametric filter.

The Raised Cosine Filter does not leak into other third-octave bands like the traditional filter. The raised-cosine filter provides a new level of precision not previously available.

3.2. Phase Comparison

The Raised Cosine Filter can be simply described as a higher order function than the traditional parametric second order section. The Raised Cosine Filter is also minimum phase in its implementation. Therefore, a Raised Cosine Filter shape may be constructed utilizing multiple minimum phase traditional parametric sections.

The following figure series (4-1 to 4-3) shows the magnitude and phase response of multiple traditional parametric sections combined to create a single Raised Cosine Filter.

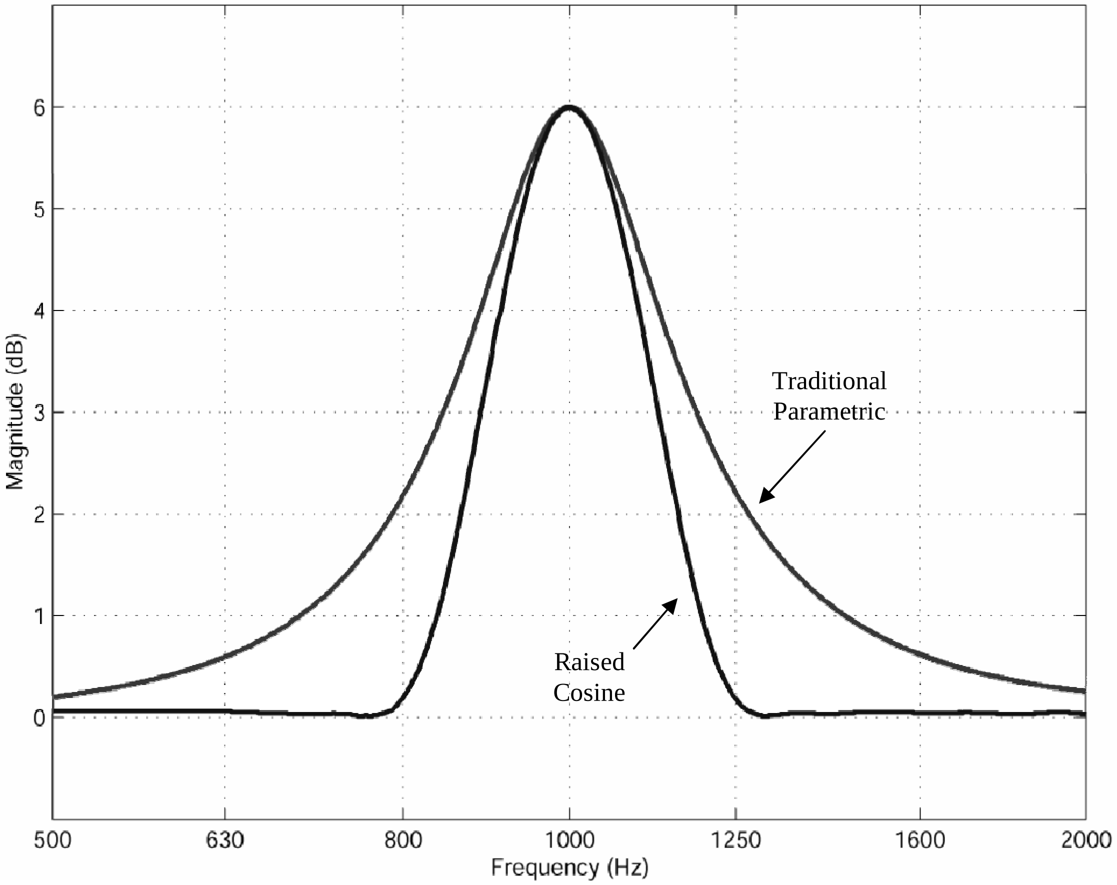

Figure 3: Raised Cosine parametric filter versus traditional parametric filter

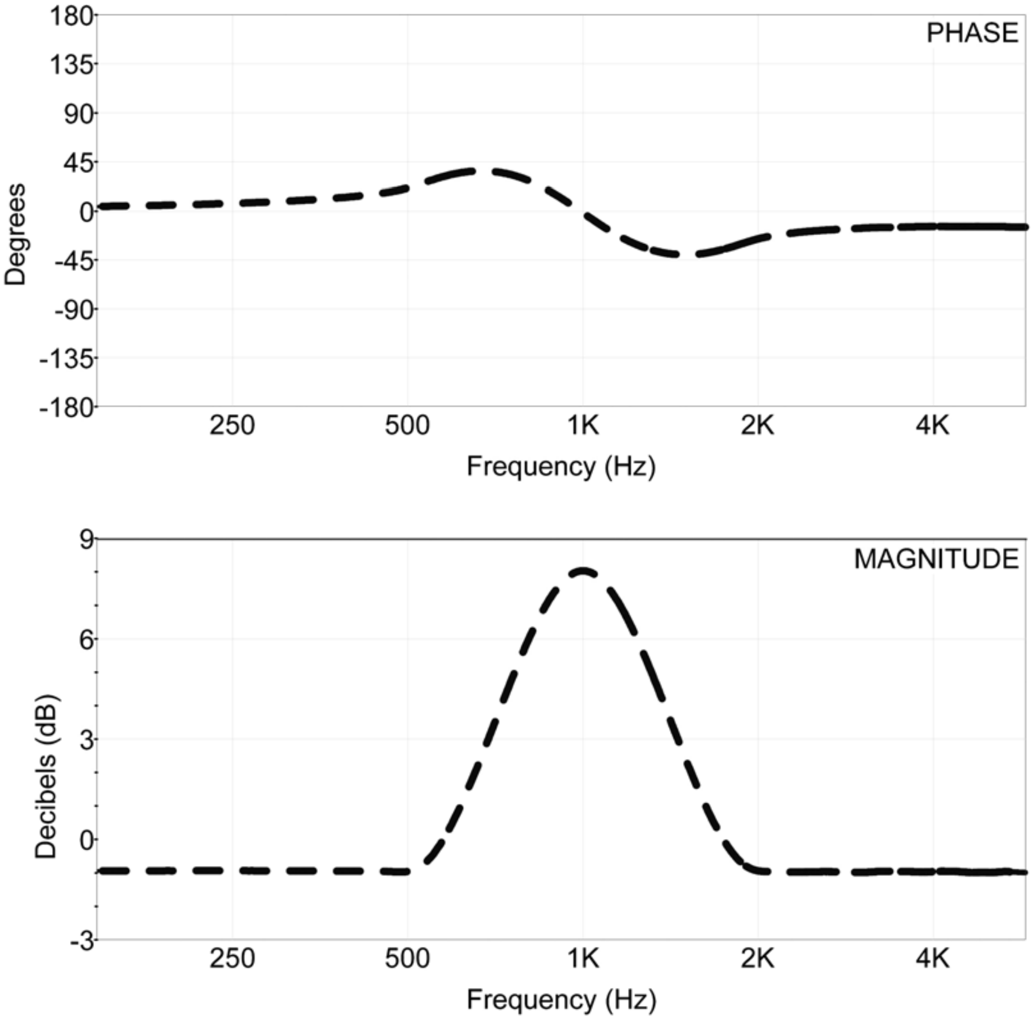

Figure 4-1: Target Raised Cosine Filter Response

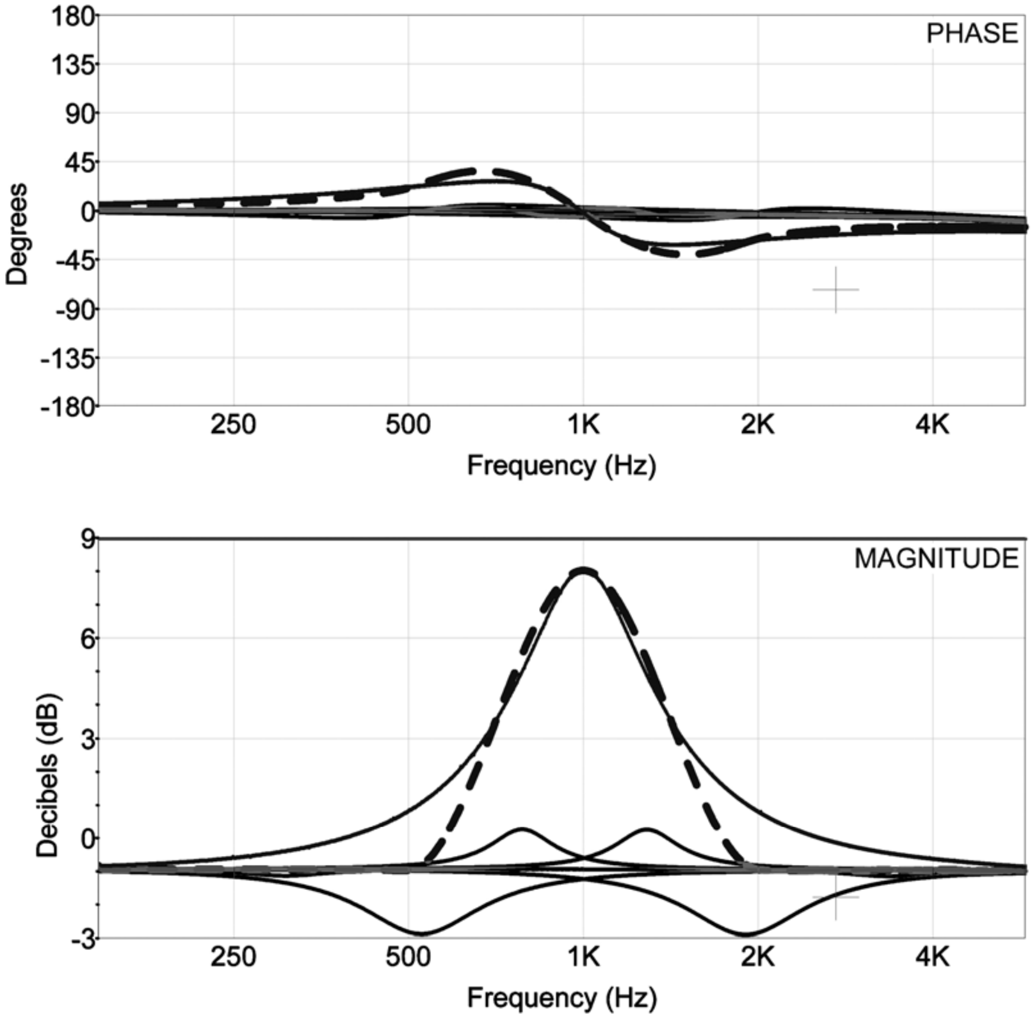

Figure 4-2: Approximation utilizing seven traditional parametric filters – individual responses shown

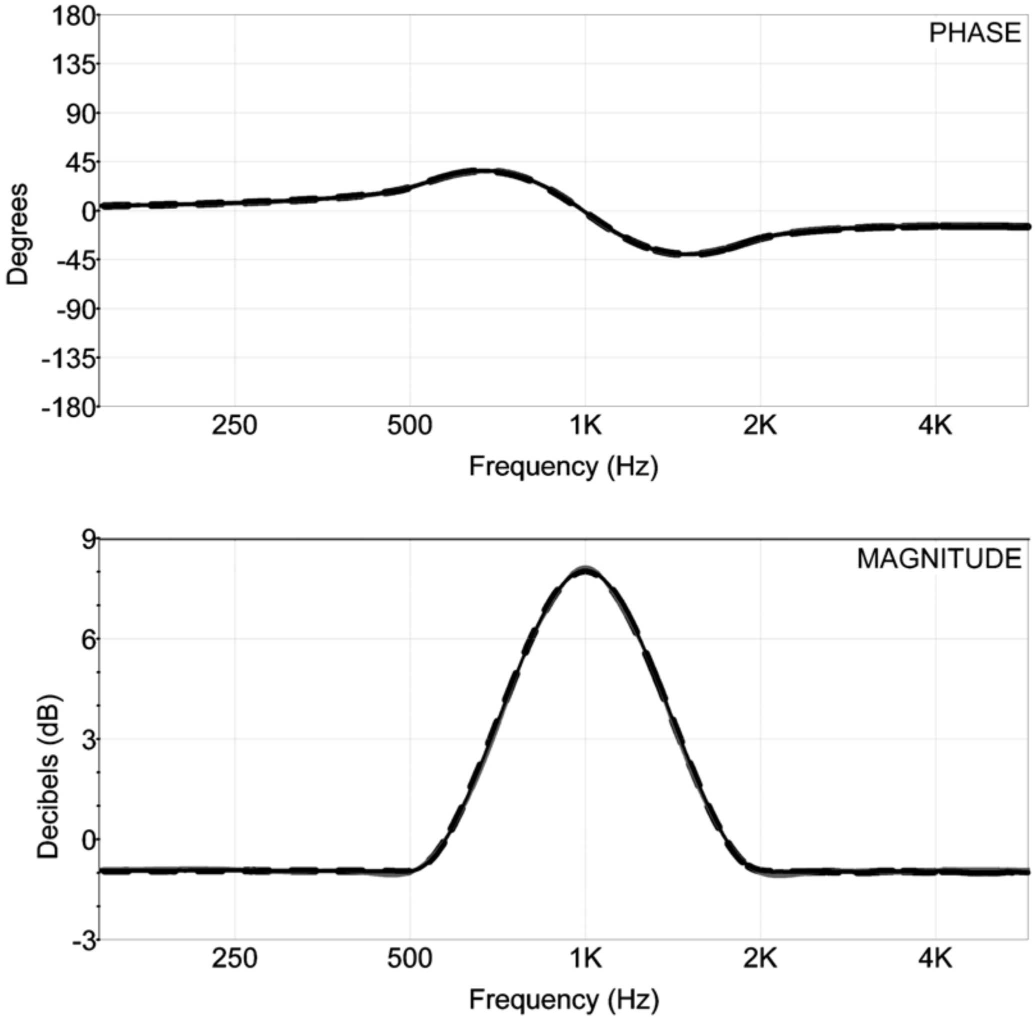

Figure 4-3: Approximation composite response, +/- 0.1 dB, +/- 1 degree tolerance

As the previous figure series shows, seven traditional parametric sections are required in order to match the response of a single Raised Cosine Filter to a tolerance of +/- 0.1 dB, +/- 1 degree.

4. MESA EQ FILTER

If we separate the rising edge of the parametric section from the falling edge, and optionally allow the transition widths of the rising and falling edges to be different, we create a new class of filter. We call this the Mesa Filter:

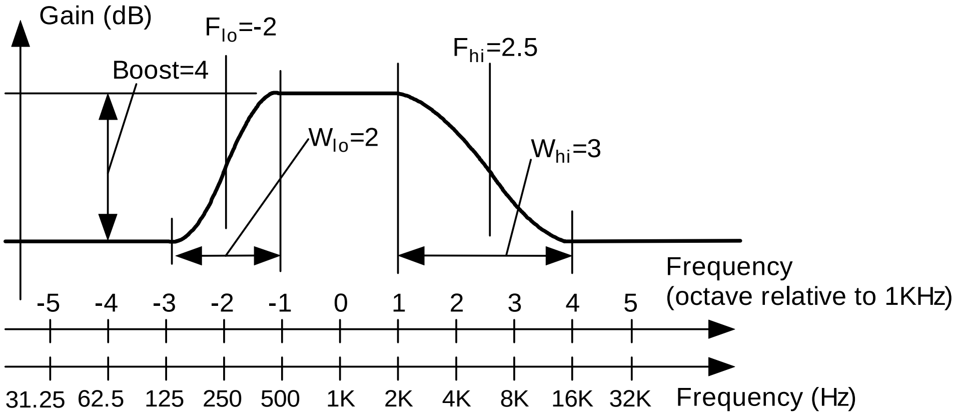

Figure 5: Raised Cosine Mesa Filter

Where:

F_{lo} :	Lower transition frequency (octaves relative to 1kHz)
W_{lo} :	Width of lower transition region (octaves)
F_{hi} :	Upper transition frequency (octaves relative to 1kHz)
W_{hi} :	Width of upper transition region (octaves)
$Boost$:	Gain of the Mesa plateau (dB)

$$Gain(f) = \begin{cases} 0 & \left(f \le F_{lo} - {}^{W_{lo}}\!/_{2}\right) \\ Boost \times \left\{0.5 + 0.5 \times \cos\left(\pi \dfrac{f - F_{lo} - {}^{W_{lo}}\!/_{2}}{W_{lo}}\right)\right\} & \left(F_{lo} - {}^{W_{lo}}\!/_{2} < f < F_{lo} + {}^{W_{lo}}\!/_{2}\right) \\ Boost & \left(F_{lo} + {}^{W_{lo}}\!/_{2} \le f \le F_{hi} - {}^{W_{hi}}\!/_{2}\right) \\ Boost \times \left\{0.5 + 0.5 \times \cos\left(\pi \dfrac{f - F_{hi} + {}^{W_{hi}}\!/_{2}}{W_{hi}}\right)\right\} & \left(F_{hi} - {}^{W_{hi}}\!/_{2} < f < F_{hi} + {}^{W_{hi}}\!/_{2}\right) \\ 0 & \left(F_{hi} + {}^{W_{hi}}\!/_{2} \le f\right) \end{cases}$$

5. NEW EQ CONTROLS

The flexibility embodied in these new filter shapes requires new methods of control. With up to six parameters to adjust, if these controls are not easily accessible to the user, the shaping tools would become time consuming to use in practical application. Rather than using the traditional metaphor of mimicking physical analog controls, we present a new graphical interface which permits easy adjustment of all six parameters in real time. The graphical interface has been developed to allow fast and intuitive control with a touch screen or stylus.

The user interface provides two basic areas of control. The main section shows the various Raised Cosine Filter elements used to create the desired frequency response, along with a composite curve that accurately displays the resultant overall frequency response. Below this display is an "EQ Tool", which consists of an envelope representation of the Raised Cosine Filter. By touching with the finger, stylus or by mouse control, the user can quickly access all parameters which describe a Raised Cosine Parametric Filter or the split asymmetric Raised Cosine Mesa Filter. Dragging the center frequency and bandwidth for each side creates entirely new families of curves.

The following figure series illustrates usage of the EQ Tool interface. Figure series 6-1 to 6-3 show the EQ Tool for the Raised Cosine Parametric. Figure series 7-1 to 7-4 show the EQ Tool for the Raised Cosine Mesa.

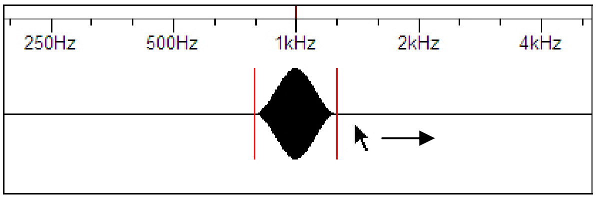

Figure 6-1: Raised Cosine Parametric - Drag outside the envelope to adjust bandwidth

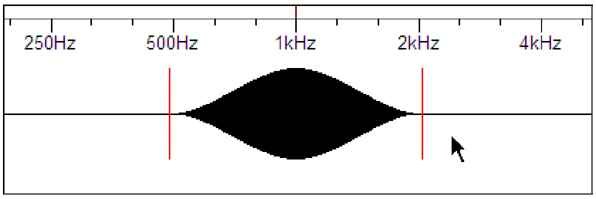

Figure 6-2: Raised Cosine Parametric - Increased bandwidth representation

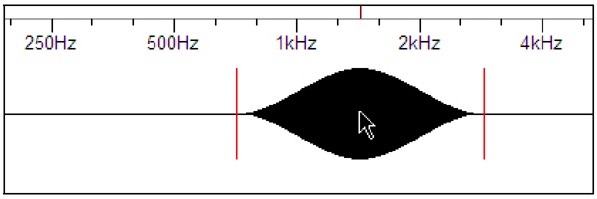

Figure 6-3: Raised Cosine Parametric - Drag inside envelope to adjust center frequency

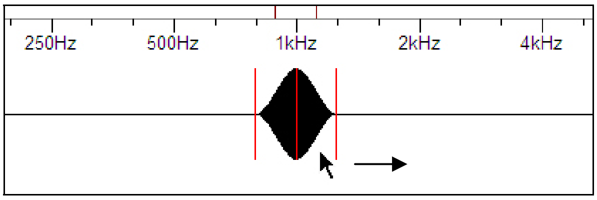

Figure 7-1: Raised Cosine Mesa – Drag one side of the envelope to adjust one center frequency

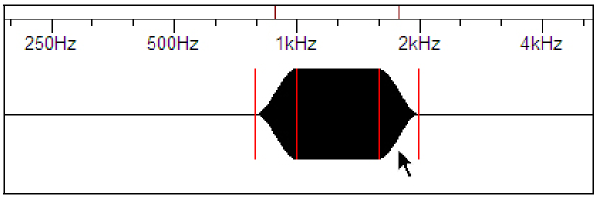

Figure 7-2: Raised Cosine Mesa – Symmetrical Mesa Filter function

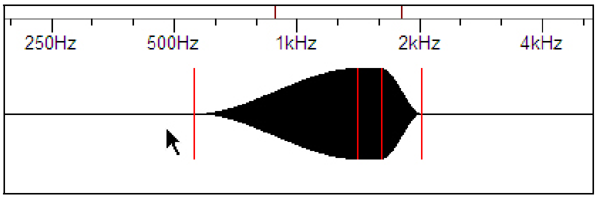

Figure 7-3: Raised Cosine Mesa – Drag outside the envelope to adjust bandwidth

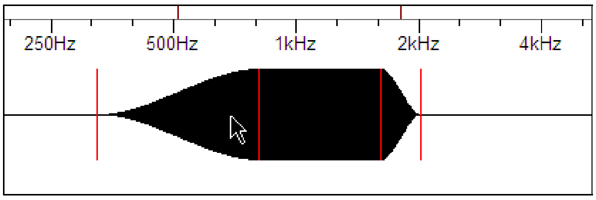

Figure 7-4: Raised Cosine Mesa – Drag one side of the envelope to adjust second center frequency

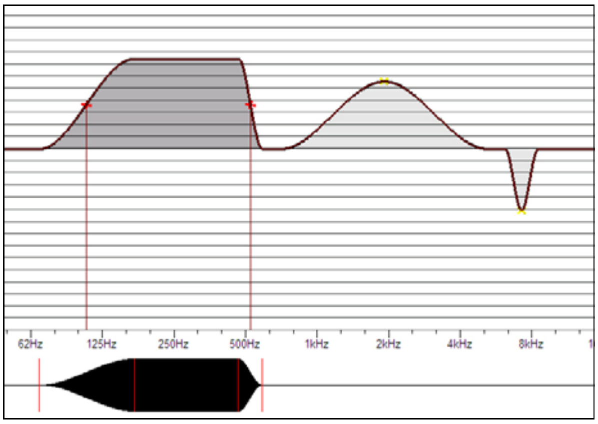

Figure 8: Family of Raised Cosine Filter shapes, showing EQ display and EQ Tool

6. GRAPHIC EQUALIZATION

Graphic equalizers get their name from the notion that you can adjust the graphic equalizer controls to create a shape that provides a desired frequency response. The visual representation of the graphic equalizer's front panel or software interface controls should reflect what is happening to the audio signal. Unfortunately this is not the case when using traditionally implemented analog and digital filters.

For example, figure 9 shows the front panel controls of an analog graphic equalizer whose sliders have been set to +6 dB from 500 Hz to 2 kHz.

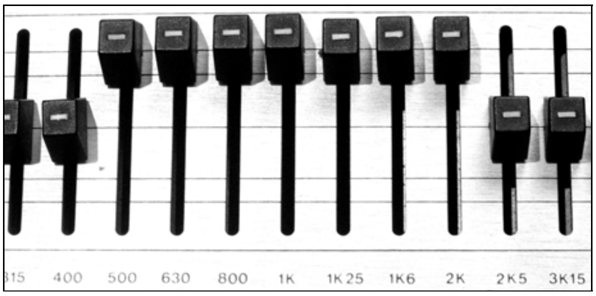

Figure 9: Analog graphic equalizer controls

Figure 10 shows the measured transfer function magnitude resulting from these slider positions.

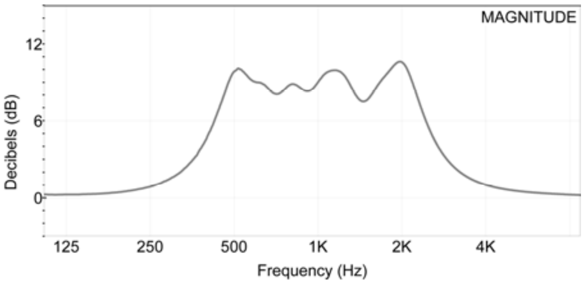

Figure 10: Analog graphic equalizer response

As Figure 10 shows, the interaction amongst the adjacent traditional parametric filters creates a non-intuitive response that results in peak magnitudes exceeding 9 dB.

Regardless of how the electrical combination of the filters in the graphic equalizer filter bank is performed, the resultant magnitude frequency response will suffer from these interaction effects. Digital graphic equalizers also suffer from these magnitude response anomalies, displaying errors on the same order of magnitude, although the ripple is more consistent through the band of emphasis.

Figure 11 shows the user interface controls of a Raised Cosine implementation of a graphic equalizer whose controls have been set to +6 dB from 500 Hz to 2 kHz.

Figure 12 shows the measured transfer function magnitude resulting from these user interface control positions, as compared to the measured transfer function magnitude resulting from the traditionally implemented analog graphic equalizer as previously shown.

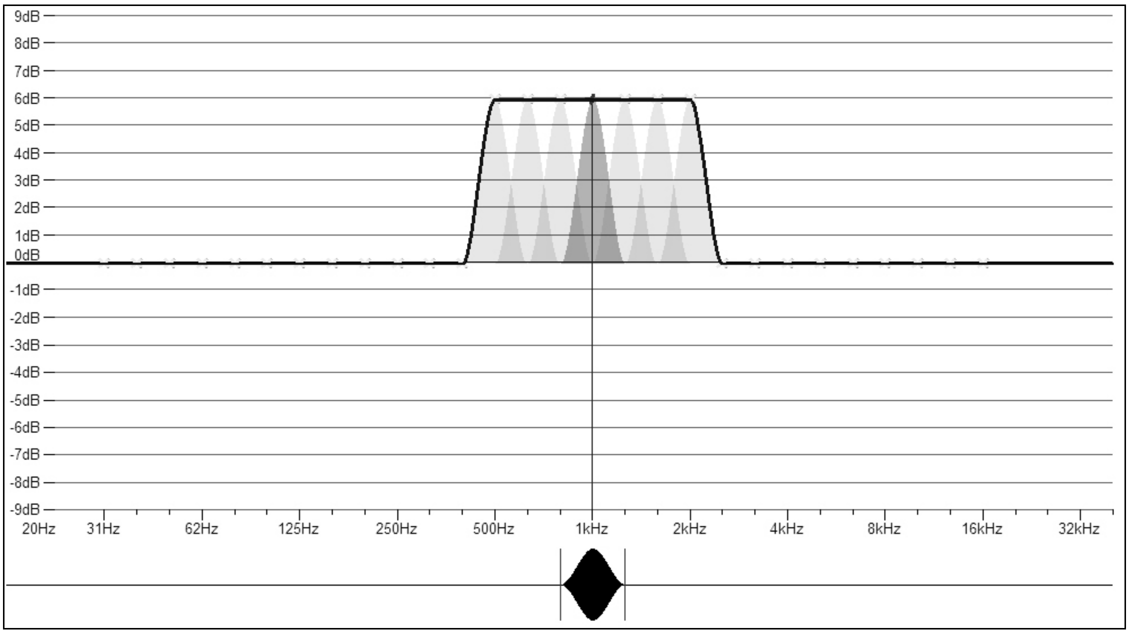

Figure 11: Raised Cosine graphic equalizer controls

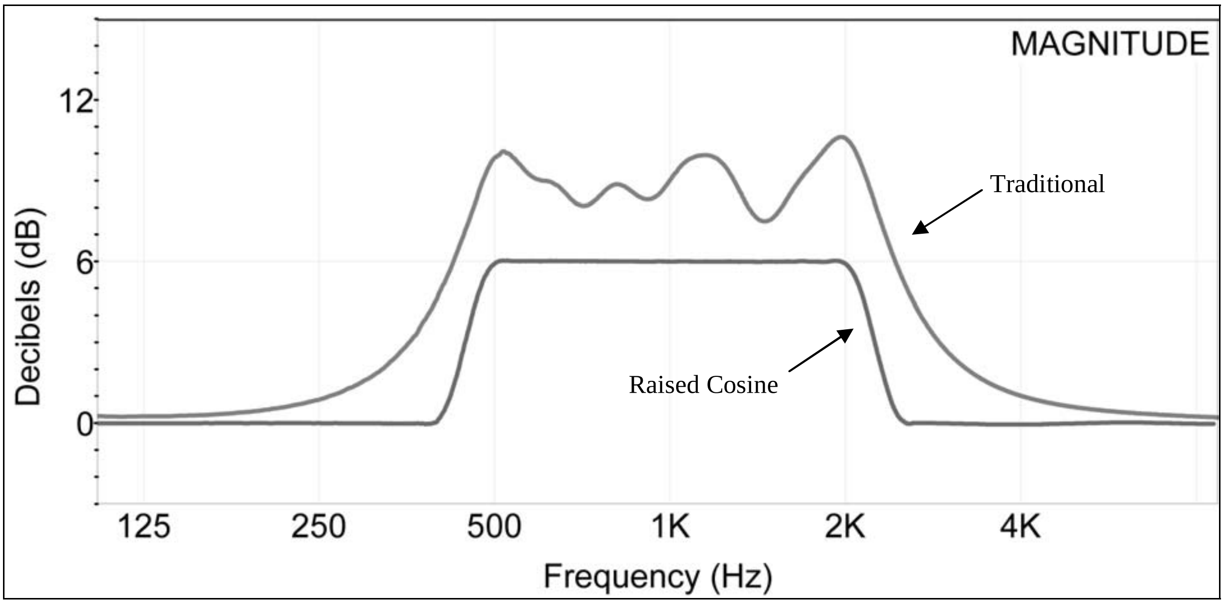

Figure 12: Raised Cosine graphic equalizer response compared to traditional graphic equalizer

The Raised Cosine Filter provides flat summation of adjacent filters regardless of the boost or cut magnitude. Flat summation will occur as long as the fractional octave bandwidth equals the fractional octave spacing between the filters. Figure 13 shows a Raised Cosine Filter typology, and Figure 14 shows a traditional parametric typology for comparison.

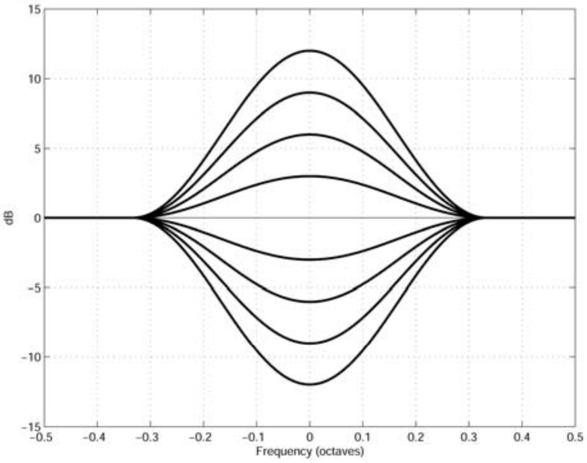

Figure 13: Raised Cosine Parametric Typology

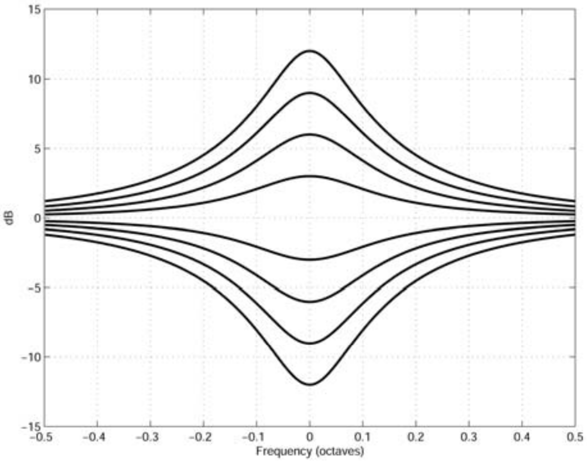

Figure 14: Traditional Parametric Typology

7. SYSTEM OPTIMIZATION

Applications of digital filtering techniques to sound system optimization are well known. A large body of work exists on both traditional and reverberation-reducing methods of room equalization. Mourjopoulos has contributed significantly to these efforts [2, 3], providing research into both minimum phase and excess phase optimization methods. Fielder [4] suggests that room acoustics optimization is limited to minimum phase equalization methods. In either case, minimum phase optimization methods are an important instrument for correcting loudspeaker-room interaction. The following example illustrates the effectiveness of minimum phase optimization.

A loudspeaker response was measured in an anechoic chamber, providing a reference transfer function measurement shown in figure 15. A complex reflector was placed in the anechoic chamber to produce a representative reflection source, shown in figure 16. Raised Cosine Filters were used to create a minimum phase optimization, shown in figure 17.

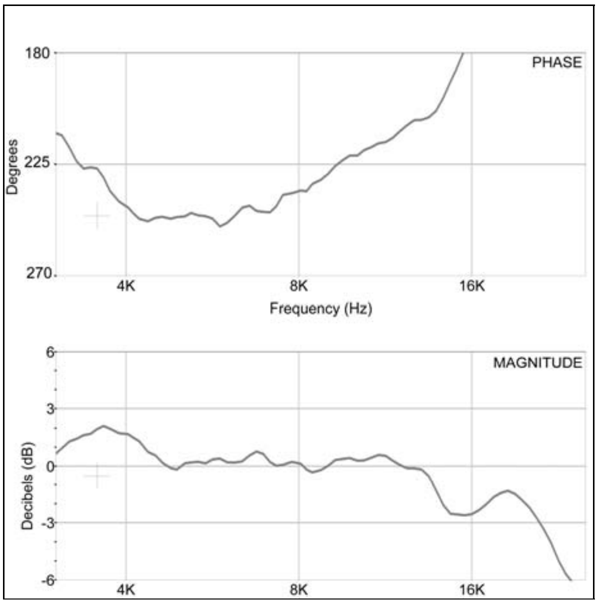

Figure 15: Anechoic response

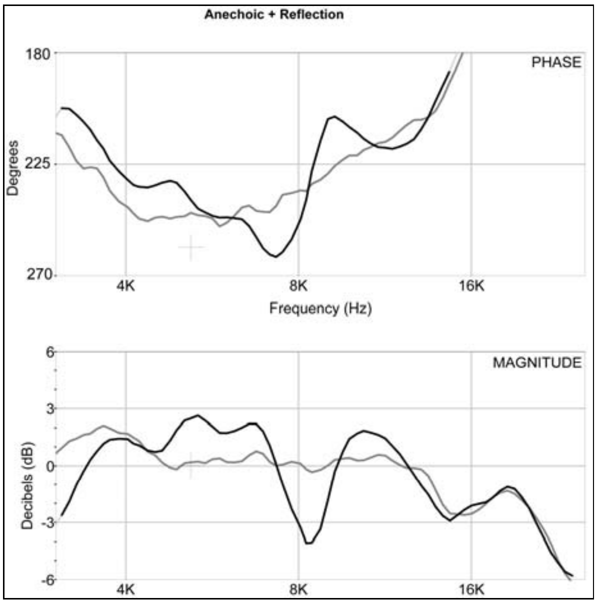

Figure 16: Anechoic and reflection responses

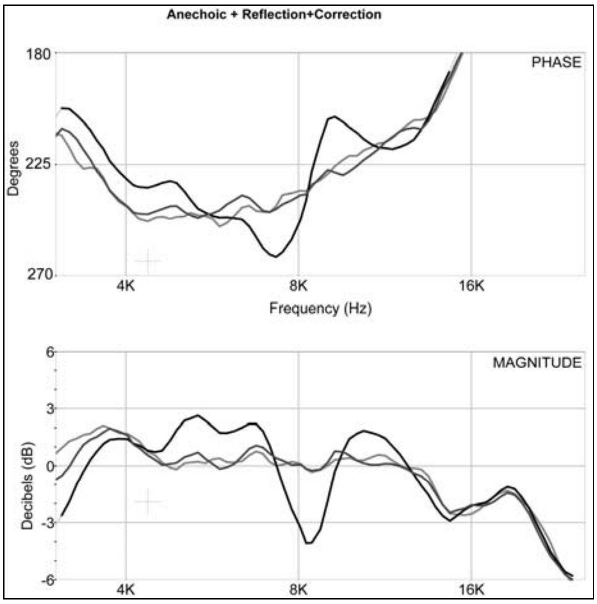

Figure 17: Anechoic, reflection and optimized responses

Figure 17 illustrates that minimum phase equalization can be effectively implemented to compensate for acoustic reflection sources. More specifically, the use of minimum phase equalization functions can correct artifacts created by minimum phase loudspeaker-room interaction.

It should be noted that fully compensating for reflections is for example purposes only; illustrating that a minimum phase equalization function is capable of compensating for loudspeaker/room interactions. In practical application, correcting amplitude response errors caused by reflective surfaces should only be performed when such an anomaly is found to be consistent across the listening area. Multiple listening positions should be analyzed prior to performing sound system optimization.

In the above example, the reflection source created a response anomaly that was reasonably symmetric. There are many interaction effects that cause more asymmetrical response functions. For example, the interaction between sound and air creates an asymmetrical response function at high frequencies. Traditional symmetric parametric filters are typically used to correct for sound attenuation due to air propagation; this result in emphasis of out-of-band high frequency energy that can increase distortion and shorten the lifetime of the loudspeaker component.

The aforementioned Raised Cosine Mesa Filter provides an improved solution to this optimization scenario. Figure 18 depicts the high frequency response of a loudspeaker array element, as measured in the listening area of a concert hall. As can be seen in the figure, high frequency absorption is exhibited. By applying a single Raised Cosine Mesa Filter, the air attenuation can be easily compensated.

Sound engineers have relied upon parametric filter banks in order to obtain a similar optimization characteristic. It is a time consuming process to incrementally tune a bank of parametric filters to provide a similar response, due to the interaction between neighboring filters.

Figure 19 illustrates an attempt to match the response of a single Raised Cosine Mesa Filter with a bank of six parametric filters. This figure represents the complexity involved in creating an optimization with traditional parametric filters.

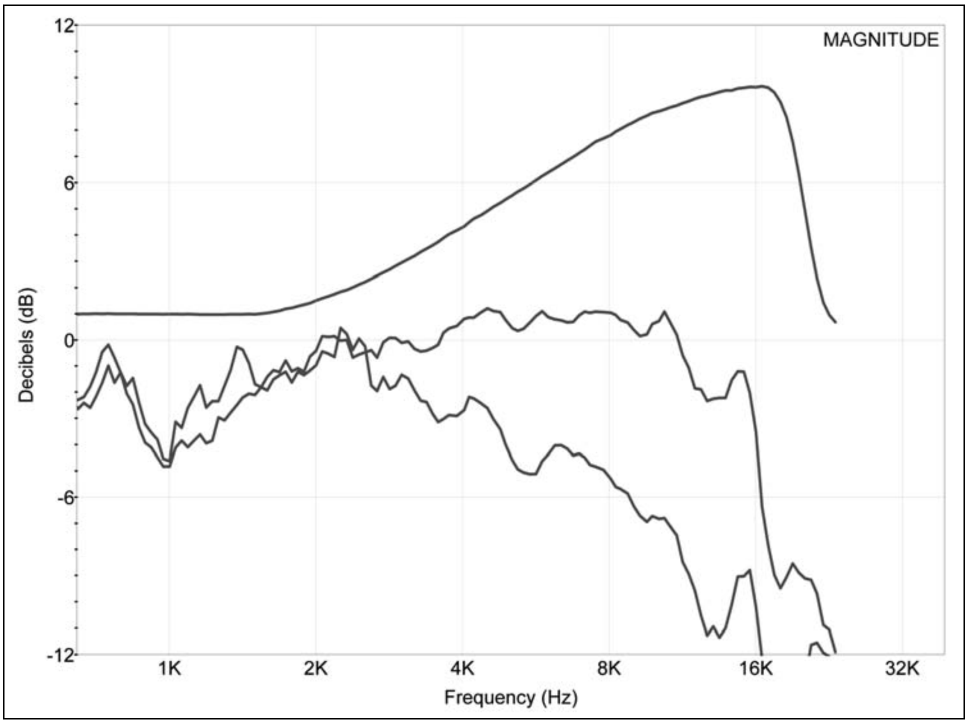

Figure 18: Raw and Raised Cosine Mesa Filter optimized high frequency response

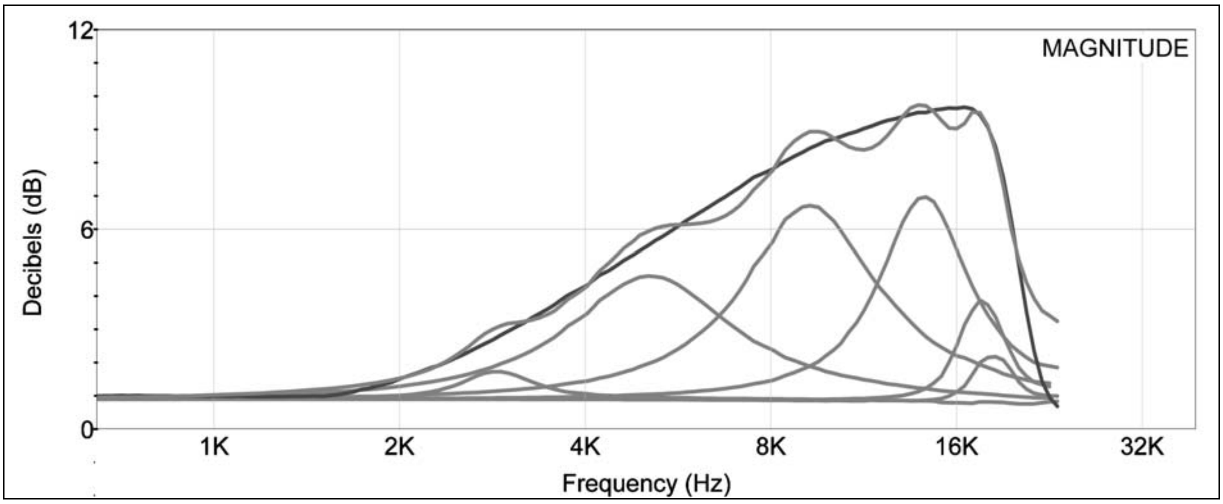

Figure 19: Traditional parametric filter bank approximation to Raised Cosine Mesa Filter

8. CONCLUSION

Since the introduction of Baxandall's tone control in 1952, we have seen a steady audio equalizer evolution from analog implementations, performing simple bass and treble tone controls, to the parametric EQ popular today. Digital consoles and outboard EQ processors generally mimic their analog counterparts. Beyond reproducing analog functions, digital signal processing offers completely new ways of contouring sound.

Through a new digital filter synthesis technique with 1/24th octave resolution, we have shown the benefits of a new filter shape based on a mathematical function called a Raised Cosine Filter. The Raised Cosine Filter has numerous beneficial attributes. In addition to providing improved selectivity, it has the benefit of perfect summation between neighboring filters, leading to an ideal response for Graphic Equalization applications.

The new Raised Cosine Mesa Filter shape resembles the table like land formations in Arizona, USA. The Mesa Filter permits rapid and precise creation of useful filter shapes such as frequency tilts commonly used with line array loudspeaker systems. Coupled with new graphical interface controls, we have demonstrated that these new shapes based on the Raised Cosine are a powerful addition to the sound engineer's toolkit.

9. ACKNOWLEDGEMENTS

The authors would like to thank Jim Meyer, Marcus Altman and Anthony Hale for their contributions to the content of this paper.

10. REFERENCES

[1] Massenburg, G., "Parametric Equalization", presentation number K-2, presented at the 42nd convention of the Audio Engineering Society, 1972, May 2-5.

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